

Answer all of the following questions. Each question counts 5 points.

Calculators are not allowed

1. Use the definition of the limit to prove that

$$\lim_{x \rightarrow \frac{1}{3}} (3x - 4) = -3.$$

2. Evaluate the following limit (if exists):

$$\lim_{x \rightarrow 0} \frac{1 - \cos 3x}{x^2}$$

3. Find the horizontal and vertical asymptotes (if any) for the graph of the function:

$$f(x) = \frac{x|x|}{x^2 - 4}$$

4. Let f be a function defined by:

$$f(x) = \begin{cases} \frac{2(x^2 - 1)}{|x - 1|}, & \text{if } x < 0, \\ Ax + B, & \text{if } 0 \leq x \leq 1, \\ \frac{\cos x + 2x - 1}{x}, & \text{if } x > 1 \end{cases}$$

Find the values of the constants A and B so that f is continuous for all values of x .

5. (a) State the Intermediate Value Theorem.

(b) Show that the equation $\cos x = 2x$ has a solution in $(-\frac{\pi}{2}, \frac{\pi}{2})$.

6. If $f(x) = \sqrt{x^2 + 5} + 7$, use the definition of the derivative to find $f'(2)$.

7. Let $f(x) = x^{\frac{1}{2}}(x-1)^{\frac{2}{3}}$. Find all points at which the graph of f has a vertical tangent line or a cusp.

8. Find the derivative of the function

$$f(x) = \sin\left(\frac{x}{2}\right) \cos(2x) + \tan\left(\frac{\pi}{4} - x\right).$$